Lecture 8 (Nathan Kutz)

Problem 1

What is a standard approach for tackling an ill-posed problem?

- **(a)** Use regularization.
- **(b)** Acquire more data.
- **(c)** Use a different coordinate system.

Note: Another common approach is to choose a different problem ;-)

Problem 2

For $x(t) \in \mathbb{R}$, we estimate the governing equations of the system:

$$\dot{x}(t) = -\sin(x(t)), \quad t \geq 0,$$

by solving the following regularized regression problem:

$$\min_{\Theta \in \mathbb{R}^5} \left\| \begin{pmatrix} \dot{x}(t_1) \\ \dot{x}(t_2) \\ \dot{x}(t_3) \\ \vdots \end{pmatrix} - \begin{pmatrix} 1 & x(t_1) & x(t_1)^2 & x(t_1)^3 & x(t_1)^4 \\ 1 & x(t_2) & x(t_2)^2 & x(t_2)^3 & x(t_2)^4 \\ 1 & x(t_3) & x(t_3)^2 & x(t_3)^3 & x(t_3)^4 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \cdot \begin{pmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix} \right\|^2 + \lambda \sum_{i=0}^{4} |\theta_i|,$$

where $\lambda > 0$ is the regularization parameter and $\Theta = (\theta_0, \ldots, \theta_4)$. The trajectory $x(t)$ is initialized at $x(0) = 0.1$ and $t_1 = 0.1$, $t_2 = 0.2$, $t_3 = 0.3$, ... Which of the coefficients θ_i are expected to be non-zero for a well-chosen parameter λ that promotes sparsity?

- **(a)** θ_0
- **(b)** θ_1
- **(c)** θ_2
- **(d)** θ_3
- **(e)** θ_4